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delft hydraulics laboratory

design and operation of movable-bed river-models

M. de Vries

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1. Introduction

In river-engineering movable-bed models are important tools as to the proper design of river improvements. Moreover the movable-bed can be used to understand the morphological phenomena involved. The schematized situation, present in the model, together with the relative ease at which the data can be collected and the comparatively large speed at which the morphological processes take place make that these models are quite useful for the extension of our knowledge in fluvial processes.

The use of the models for practical design purposes as well as for back-ground research is evident. However, the real problems are present in the prototype! Relatively little effort is made in comparing the model-results with the results later on obtained in the prototype. Moreover morphological studies in models require extention in the prototype. It can be stated that the use of models does not reduce the need of prototype data: on the contrary, in practice more prototype data are required when a particular riverreach is modeled. Another general remark has to be made. Movable-bed models are only tools albeit important ones. The skill of the investigator is important for the design, the operation and the interpretation of the model.

Here, rather than giving a complete course on modeling two aspects will be treated here particularly. Design and operation will be discussed in their mutual relation.

2. Determination of scales

2.1. General

The geometric reduction which takes place in modeling causes that also non-geometric properties (velocities, densities etc.) have to be scaled. The determination of scales consists of finding a set of scales for all important parameters in such a way that the required degree of similarity can be expected. This is the reason why "theories on hydraulic models" (Yalin, 1971) only have restricted value in finding a compromise between conflicting conditions for the various scales. The question is not how to build a model without scale-effects; the problem rather is how to compromize for a particular case in order to reduce the errors due to scale-effects to an acceptable magnitude. This brings forward the economical aspects in the use of models in engineering in general.

In order to facilitate communication some definitions are necessary. As definition of the <u>scale</u> (or scale-factor) the ratio between prototype value and model value will be used.

Thus

$$\text{length scale} = n_L = \frac{L_p}{L_m} \qquad (1)$$

The advantage is that most scales will be larger than one, which facilitates mental arithmetic.

Moreover we need a definition of a scale-effect. Scale-effects are said to be present if the scale of a parameter is not a constant but varies in space and/or time. In practice this becomes apparent if the scale of a parameter is not only a function of <u>scales</u> of other parameters but also of the parameters themselves. As an example Fig. 1 shows the variation of n_H, the scale of the energy head (n_H) as a function of depth scale (n_H) and the scale of the velocity-head (n_s). Outside the point for which $n_H = n_s$ also the parameter Fr_m, which can vary over the model is of importance. This example can easily be deduced from Bernoulli equation

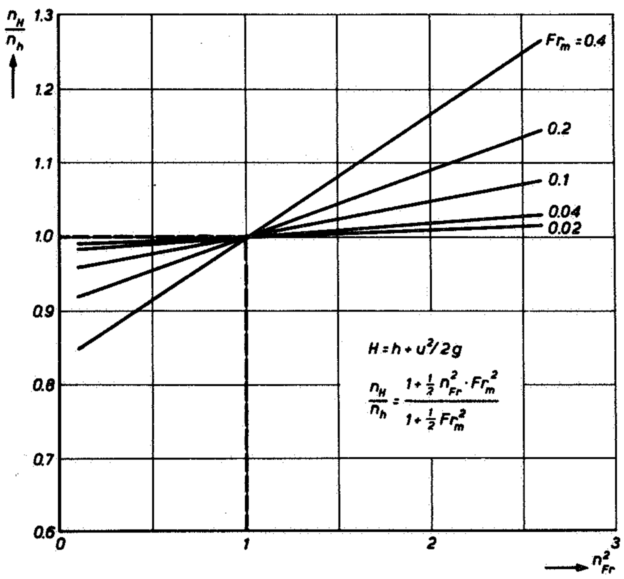

Fig. 1 - Froude condition; scale effects

$$H = h + u^2/2g \qquad (2)$$

and it shows that scale-effects are present if Froude condition $(n_{Fr} = 1$ or $n_u = \sqrt{n_h})$ is not fulfilled.

The determination of suitable scales for a particular problem has to be carried out from the knowledge of the physical process involved.

From these physical processes scale-laws and scale-conditions can be deduced. It is of importance to distinguish here. Scale-laws are fulfilled automatically and therefore have to be considered as a must for the scaling procedure. Scale-conditions are requirements that have to be fulfilled in order to avoid scale-effects. Two simple examples will clarify the difference.

(i) Froude condition expresses for free-surface flow a link between depth scale and velocity scale, deviation from this condition (sometimes necessary and acceptable) leads to scale-effects.

(ii) Physical laws are valid in model and prototype. The scale-laws derived from these physical laws express a must. This is for instance the case for Chézy-equation

$$u = C\sqrt{hS} \tag{3}$$

or

$$m_u = n_C \cdot n_h^{\frac{1}{2}} \cdot n_S^{\frac{1}{2}} \tag{4}$$

This scale-law (or a similar expression based on Manning equation) cannot but be fulfilled.

The task of proper scaling now consists in finding scale-relations (i.e. both scale-laws and scale-conditions) and find by compromizing a suitable set of scales which fulfils the requirements set for a particular case.

How to find these scale-relations? In principle two ways are possible:

(i) By the procedure of dimensional analysis and

(ii) By considering the physical-mathematical description of the problem.

The following remarks have to be made

ad (i) Dimensional analysis consists of guessing the parameters important to the problem at hand, followed by a systematic procedure to derive dimensional products (c.f. Langhaar, 1957). This procedure is based on the general properties of dimensions. No information on the magnitude of scale-effects is obtained.

ad (ii) From the physical-mathematical description scale-relations and the magnitude of possible-effects (c.f. Fig. 1) can be derived. The accuracy of the result, however, depends entirely on the validity of the basic equations!

2.2. Scale-laws and scale-conditions

For mobile-bed river-models various scale-relations have to be fulfilled at the same time. The morphological process is characterized by a large interaction between watermovement and sedimentmovement. It is necessary to model this morphological process carefully thus both watermovement and sedimentmovement have to be considered.

First of all Froude-condition has to be fulfilled for the free-surface flow present (c.f. Fig. 1). Secondly roughness-condition has to be considered. For river-studies the curvature of the streamlines has to be reproduced properly. Otherwise the bedconfiguration, result of an interaction between the watermovement and the sedimentmovement will be wrong. The curvature of the stream-lines is governed by (i) the presence of sidelong guidance of riverbanks and (ii) by the bed-roughness. The first cause is reproduced properly if geometric similarity with regard to the planform is present. The influence of the bedform is reproduced properly if roughness-condition is fulfilled (Bijker et al, 1957).

$$n_C^2 = \frac{n_L}{n_h} = r = \text{distortion} \qquad (5)$$

or a similar expression if not Chézy equation is used.

It can be remarked that if Froude condition and roughness-condition are fulfilled, then automatically the slope is reproduced correctly as can easily be seen from Eqs. 4 and 5 and Froude condition yields

$$n_S = \frac{n_L}{n_h} = \text{distortion} \qquad (6)$$

For rigid-bed models their is usually no problem to achieve this result. However, for movable-bed models also the rquirements for the sedimentmovement have to be considered. It is therefore also necessary to ask what happens if roughness-condition is not fulfilled correctly. From Fig. 2 it can be seen how the scale of the radius of curvature of the streamline (n_R) deviates from length-scale if n_C is not selected properly:

- the curvature of the flowlines is too strong if the model is too rough
- the curvature of the flowlines is too small if the model is too smooth.

To what extent deviation from the correct Chézy scale can be accepted depends entirely on the problem. For rivers with a large

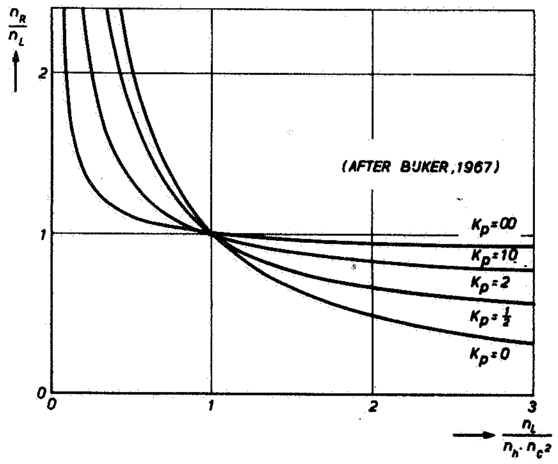

Fig. 2 - Roughness condition; scale effects

depth/width ratio the flow lines will mainly be governed by the guiding banks, roughness condition is then not very important. However, for a small depth/width ratio the bedroughness mainly governs the flow pattern and thus roughness-condition is important.

In considering the sediment movement, first of all the morphological time scale has to be considered. If the sediment transport (q_s) is expressed in apparent volume (i.e. including pores) then a coefficient of porosity does not appear in the equation of continuity of the bed material

$$\frac{\partial z}{\partial t} + \frac{\partial q_s}{\partial x} = 0 \qquad (7)$$

This equation is valid if the change of the amount of sediment in suspension can be neglected with respect to the other terms. The morphological time scale is then expressed as follows

$$n_t = \frac{n_L \cdot n_h}{n_{q_s}} = \frac{n_L^2 \cdot n_h}{n_{Q_s}} \qquad (8)$$

Where Q_s is the sediment transport over the entire width (b) and $n_b = n_L$ is a logical choise. Usually the morphological time scale (n_t) is different from the time scale ($n_{t'}$) for the watermovement

$$n_{t'} = n_L/n_u \qquad (9)$$

Selecting $n_t = n_{t'}$ gives a "slow" model (small time scale) and puts a restriction to the transport scale ($n_{q_s} = n_u \cdot n_h$) which is an extra condition for the selection of the bed-material in the model.

To obtain similarity in the morphological process it is essential that the morphological time scale is a constant in space; it is not always necessary to have n_t constant in time. From Eq. (8) it can be seen that only if n_{Qs} is a constant in space is called the ideal velocity scale (Bijker et al, 1957).
This important condition implies that for morphological models the velocity scale usually deviates from Froude condition.

It is of importance to realize that our knowledge on sediment transport is restricted. Therefore modeling based on a particular sediment transport formula has to be avoided unless it is evident that this formula is valid for the problem considered. It is advised to base the scaling on general knowledge about the parameters governing sediment transport. For prevailing bed-load transport (i.e. roughly $u_*/W < 5/3$) most formulae can be expressed as the general function

$$\frac{q_s}{D^{3/2} \sqrt{g\Delta}} = F\left\{\frac{\Delta D}{\mu h S}\right\} \tag{10}$$

in which $\Delta = (\rho_s - \rho)/\rho$ is the relative density of the bedmaterial and μ (= ripple factor) expresses somehow that only part of the shear stress is used for sediment transport. The condition following from this general relation reads that scale effects are avoided if

$$n_{q_s} = n_D^{3/2}\, n_\Delta^{\frac{1}{2}} \tag{11}$$

and

$$n_{\mu h S} = n_D \cdot n_\Delta \tag{12}$$

Equation (11) shows that n_{q_s} is indead constant (if possible lack of similarity on grain sorting is neglected). Equation (12) can be worked out as a condition for the ideal velocity scale (c.f. Bijker et al 1957 and Bijker, 1960).

Deduction of possible scale effects when deviating from Eqs. (11) and (12) is of course not possible as has been stated before. Therefore a specific transport formula has to be selected. Figure 3 shows the possible scale-effects for Meyer-Peter and Mueller (1948) formula, valid for coarse material (say D>0.5 mm).

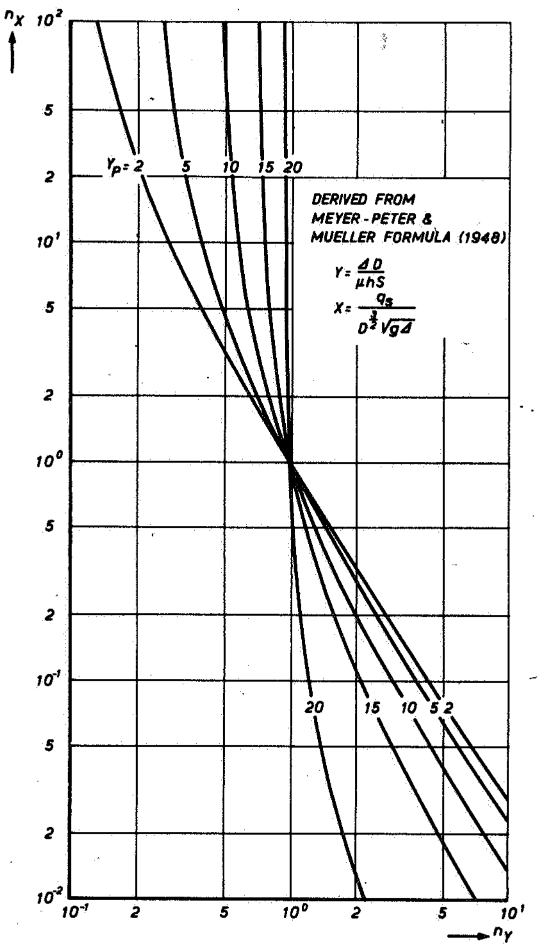

Fig. 3 - Scale effects from M.P-M formula

Neglecting for the time being the influence of C and μ, it follows from Eq. (12) and Chézy-equation

$$n_u^2 \approx n_D \cdot n_\Delta \qquad (13)$$

If sand is used in the model ($n_\Delta = 1$), $n_h = n_u^2 \approx n_D$ would mean that ideal velocity scale and Froude condition could be fulfilled simultaneously by selecting n_D equal to n_h. However, this is only true for very coarse sediment in the prototype. Otherwise the model should contain fine silt which would mean that $u_*/W < 5/3$ was not present in the model. Some relief can be found by selecting a relative light material in the model ($n_\Delta > 1$). But here the availability and notably the cost hamper an integral utilization. Moreover in many cases Froude condition is not too strict.

If suspended load becomes predominant then u_*/W should be kept equal in model and prototype, which combined with Froude condition has given useful solutions in some cases (c.f. Zwamborn, 1966). It has to be noted that the condition

$$n_{u_*/W} = 1 \qquad (14)$$

is similar to Eq. (12) if relatively coarse material is present (fall velocity according to Newton law).

Special attention has to be paid to the hydraulic roughness. For movable-bed models selection of $n_{\Delta D}$ and n_h leads to a required roughness-value. The problem is that in the design-stage this roughness is not yet known. Experimental data like the data of Guy, et al (1966) for sand can give a useful guidance.

Now, however, one of the most crucial aspects in mobile-bed river models has to be considered. Consider the scale law following from Chézy equation (Eq. 4). According to the definition of slope

$$n_S = n_{\Delta H}/n_L \qquad (15)$$

in which ΔH stands for the difference in (energy) head between two points at distance L.

For a rigid-bed model Froude condition and roughness condition can be fulfilled and Eq. (14) and (15) lead to the desirable result

$$n_{\Delta H} = n_h \qquad (16)$$

meaning that the waterlevels are reproduced correctly.

However, for movable-bed models a selection $n_u < n_h^{\frac{1}{2}}$ is frequently necessary to reproduce properly the morphological process. It can be shown easily that

$$n_{\Delta H} = n_u^2 \qquad (17)$$

when roughness condition is fulfilled properly. Thus in this case the waterlevels are not reproduced correctly. The investigator uses an artifice here by tilting the model. This implies that every level is measured from a sloping datum line (angle S_t) according to (see Fig. 4)

$$S_t = S_p \left[\frac{n_C^2 \cdot n_h}{n_u^2} - r \right] \qquad (18)$$

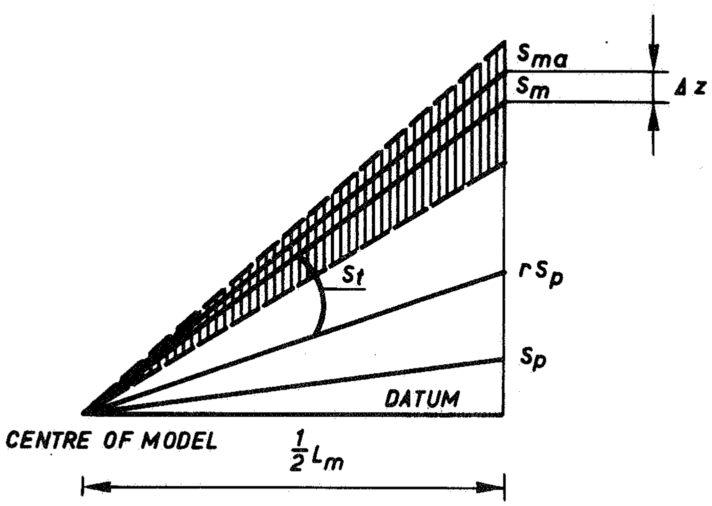

Fig. 4 - Principle of tilt

(Note that $S_t = 0$ if both Froude condition and roughness are fulfilled). For models with rigid banks or groins these parts are built from the sloping datum line. Especially in this case the value of S_t has to be known before the construction of the model. Any difference of the actual C-value in the model (C_m) from the estimated value C_m before the construction, leads to an error in the waterlevel in spite of the tilt. This leads to a slope $S_{ma} \neq S_m$. In a model reproducing a riverreach L_p the maximum error in the waterlevel ($\pm \Delta z$) will be present at both ends of the model (see Fig. 4)

$$\Delta z = \tfrac{1}{2} L_m \left[\frac{1}{C_{ma}^2} - \frac{1}{C_m^2} \right] \frac{u_m^2}{h_m} \tag{19}$$

This leads to a restriction with respect to the riverreach that can be reproduced without too large errors. According to the writers experience a length L = 10 km has to be considered as a maximum even if a fair estimate (C_m) of the actual Chézy-value of the model (C_{ma}) can be given in advance.

3. Design and operation

3.1. General

After having carried out the principle scaling a number of questions arise:

- Will only be the mean grain size be reproduced or is it necessary to reproduce "the" grain size distribution?
- What is the proper length of the river reach to be reproduced?
- Will the model be operated with a constant discharge (a somehow defined "dominant discharge") or has the regime to be reproduced?
- How is the sediment supply to the model organized and how is the sediment discharge at the downstream end measured and collected?
- How to measure in the model and how to arrive at a minimum effort to relevant conclusions, considering the errors in the measurements?

These and other questions can be and should be raised in order to arrive at good results. However, before these questions are considered in more detail, some remarks should be made on the required prototype data.

3.2. Prototype data

Any model notably a movable-bed model needs a calibration. In the design a great many uncertainties are present which cannot be checked separately. Therefore an integral check (calibration) should be carried out. The question is not whether the model is correct but to what extent is the model reproducing the prototype correctly!

This requires data from the prototype which (the model investigator should realize this) requires a large effort, but they are essential. Calibration by means of prototype data in fact means obtaining quantitative information on the scale effects present. The following example demonstrates scale-effects in the bed-topography. About 15 years ago the Delft Hydraulics Laboratory carried out a model study on a reach of the Lower-Rhine in the Netherlands. The experiments showed that for the existing situation deep spots as well as shallow spots were reproduced at the right places. However, the shoals were relatively too shallow and the troughs were relatively too deep.

The phenomenon could be quantitatively explained from the fact that the available bed-material for the model (crushed bakelite) was too uniform to reproduce the grain-size distribution of the prototype. Therefore relatively too little coarse sediment was available for the outer bend (which consequently became too deep) and too little fine sediment was available for the inner bands (which became therefore too shallow).

In reality this means that n_h varies from place to place, indicating that scale-effects are present. In practice the problem could be solved as sufficient prototype data were available. From prototype data and model data by regression analysis it could be shown that

$$n_h = f\left\{ n_{\bar{h}} \, , \, h_m \right\} \qquad (20)$$

or more specifically

$$h_p/\bar{h}_p = \alpha \left\{ h_m/\bar{h}_m \right\}^{\beta} \qquad (21)$$

with $\alpha \approx 1.03$ and $\beta \approx 0.48$ for $0.2 < h_m/\bar{h}_m < 3.0$

For the interpretation of the model results for future situations these calibration curves could be used. Note that according Eq. (21) the cross-slopes in the model were about two times too steep after the distortion had been taken into consideration. In this way rather than to operate with expensive bakelite, specially ground according to a specific grain-size distribution, a practical solution was obtaine.

This brings forward the question how the grain-size distribution should be reproduced in cases for which reproduction of the mean grain-size is not sufficient. This problem can be discussed by means of an example. In the case of a confluence or a bifurcation in fact these rivers have to be modeled at the same time. As each river has its own characteristics which is reflected in the scale relations, this scaling here is rather complicated. This is due to the fact that the three branches should obviously have some scales in common (e.g. n_L, n_h, n_u, etc.). How should the diameter scale n_D be selected?

In the design of the bifurcation of the Rhine near Pannerden, by the Delft Hydraulics Laboratory the following reasoning was taken.

1. Any deviation from "the" grain-size distribution of the upstream river (here the Upper-Rhine) will lead to deviation of the grain sorting at the bifurcation. Hence the grain-size will not be correct for the two downstream rivers and consequently their depths will not be correct. Consequently "the" grain-size distribution of the Upper-Rhine had to be reproduced correctly.
2. How can "the" grain-size distribution of the prototype be determined? The only answer is that a large number of samples have to be taken as in a normal river grain sorting is present due to the bends and the bed form. Moreover each sample should be so large that it is representative for the local grain-size distribution (de Vries, 1971). Figure 5 shows some grain-size distributions obtained in this way. Obviously "the" grain-size distribution can be defined only roughly.

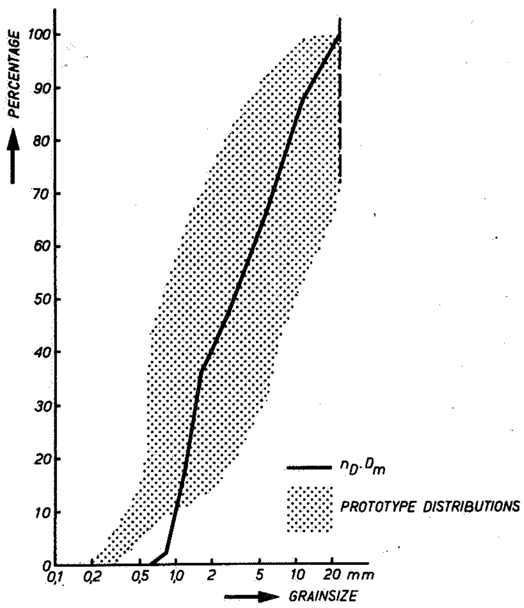

Fig. 5 - Grain-size distributions

The above given two examples regard the selection of the bed-material in relation to the availibility of prototype data. As a last example the influence of prototype data on the choise of the length of the reach to be reproduced can be discussed. Usually it is not difficult to establish the net length to be reproduced. However, also as extra upstream part as well as an extra downstream part have to be built in the model.

1. The upstream part should have sufficient length in order to be sure that in the real reach of interest(test reach)the bed topography is correctly reproduced.
2. The downstream part has to be added in order to check if the improvements of the riverreach under study do not disimprove the conditions downstream.

For the upstream part "sufficient length" should be taken. However, what does this mean? If a number of bends are added upstream then it is more or less sure that the test reach will have proper upstream conditions (i.e. independent of the distribution of water and sediment over the width at the beginning of the model). Two bends for the upstream part seems a bare minimum. However, the whole model can then become unmanageable due to the uncertainties about the hydraulic roughness (see par. 2).

Prototype data with regard to velocity distribution and sediment-transportation can help here. Introduction of these distributions in the model can lead to reduction of the length of the upstream part.

In general it can be stated that the model is not a substitute for prototype data. On the contrary, a movable-bed river-model will have results with sufficient quality if a good amount of prototype data can be made available.

3.3. Selection of bed-material

The selection of the bed material for the model is one of the most important aspects in design and operation. The following remarks can be made.

a. Very light material such as polysterene ($\rho_s \approx 1050$ kg/m^3) is expensive. Moreover it usually is rather uniform which makes it not easy to use for rivers with a large range if grain sizes. However, in cases in which the velocity scale should be in accordance with Froude condition it is unavoidable. Such an example is described by Giese, et al (1972) for a tidalriver.

This very light material adds problems to the operation of the model. First of all the collection of the bed material at the downstream end usually cannot be carried out by a sediment trap. Special devices are necessary to separate water and polysterene. Photo 6 shows the rotating device used by the Delft Hydraulic Laboratory. The water/sediment is led

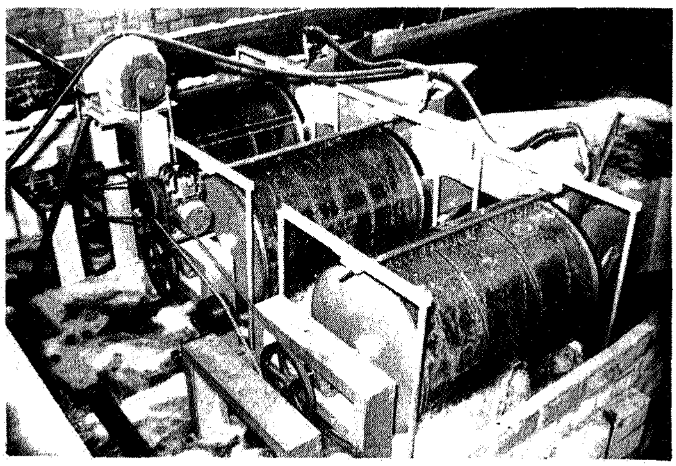

Fig. 6 - Polysterene separator

in the drum. The water leaves the installation through the holes in the outer cylinder. The sediment falls by gravity in a small flume inside the drum.

Another aspect of this very light material regards the sounding of the model-bed. The difference in density between water and polysterene is small thus manual sounding leads to errors: the bedlevel cannot be defined. Electronic equipment becomes necessary, for instance the profile-indicator used by the Delft Hydraulics Laboratory (D.H.L. 1970). A vertical needle can be steered by a servo mechanism at a distance of $\frac{1}{2}$ to 2 mm(adjustable)from the bed. The difference in conductivity between water and sediment causes that the needle remains exactly at the adjusted distance from what the instrument defines to be the bed. Moving the instrument in horizontal direction provides measuring of a cross-section.

It is obvious that the very light materials are not only expensive due to the direct costs but also due to additional equipment required to operate models with such materials.

b. Light material ($\rho_s \approx 1200$ to 1600 kg/m^3) such as ground bakelite, coal, pumice sand etc. do give a good solution in several cases. The problems of collecting the sediment at the downstream end of the model are not more serious than when sand is used as a bed material. Also the problems regarding the sounding do not require sophisticated instrumentation. The costs of the material may not be extremely high; however, the material may become expensive if a certain grainsize distribution is wanted. Some remarks should be made in addition. For ground bakelite one has to be sure that all grains have (almost) the same density. Coal is attractive as long as a hard coal is used (Zwamborn, 1966), otherwise the grainsize is reduced while operating the model for a long time. Moreover a soft coal implies that the water remains black! Pumice sand has only apparently a small density as the reduced density is only caused by the presence of pores in the grains. This implies that the "volume-weight" depends on the grain-size.

c. <u>Sand</u> ($\rho_s \approx 2650\ kg/m^3$) is widely used for movable-bed river-models. In principle it is not expensive. This is, however, the case unless special requirements are set with regard to the grain-size distribution of the bed-material in the model (c.f. Fig. 5). Some times this can lead to the use of special devices in order to reach a bed-material of a prescribed grain-size distribution.

3.4. Reproduction of the river-regime

The reproduction of the regime of the river requires special attention. The strongest schematization of the regime is obtained when the model is operated with a constant discharge. It is easier to find a name for this discharge ("dominant discharge") than to indicate what this discharge represents; leaving aside how this discharge should be determined!

First of all it should be emphasized that rivermorphology deals with time-depending processes. One single discharge therefore is not able to reproduce these processes. The only goal that can be reached is that a reasonable average situation can be obtained.

In the discussion of the reproduction of the regime first of all the difficulties met in introducing the entire regime should be mentioned. In chapter 2 the scaling for one single discharge is treated. In principle this can be carried out for a number of discharges characterizing together the regime. However, now the following conditions should be considered.

(i) For each discharge the ideal velocity scale should be selected, which leads to a transport-scale (and time scale) which is not time-dependent, provided no grain-sorting is present.

(ii) As the tilt is bound to the construction of the model, S_t should not vary with the discharge.

In many cases these conditions conflict. For relatively long rivermodels the second condition is important and then time scale (and transport scale) are allowed to vary. Consequently the discharge hydrograph of the river has to be introduced on a varying time scale. This artifice together with the relatively complicated control system for discharges and waterlevels has in many cases led to the question whether by using one single discharge not an equally accurate result could be obtained for the model.

Only little experimental evidence is available on data obtained by both introduction of a regime and a single discharge. It seems that the study by Ackers and Charlton (1970) on meandering is an exception. However, also this study ment to define experimentally a dominant discharge for the meander characteristics does not help in the problem mentioned here.

It can be stated that results can only obtained after adequate computational methods for bed-level variations with two space dimensions have been developed.

Regarding the dominant discharge it has been stated already that only a rough average situation can be reproduced. It has to be expected that each river characteristic (bed level, minimum depth, meander length) will require its own dominant discharge. This can directly be indicated by arguing that the morphological processes are highly non-lineair. Therefore a definition of the dominant discharge as the one carrying the average yearly sediment transport is a blind guess. Why should a "year" for the schematized river with constant discharge be of equal length as the year for the real river?

Following this line of thinking it has been stated that computations should give information on time-depending fluctuations of relevant parameters such as bed levels (Prins and de Vries, 1971). This information can then be used to define the discharge that alone can reproduce a value of a parameter (mean bed-level or minimum depth) that is relevant to the problem. This demonstrates the close cooperation between both mobile-bed river-models and mathematical models of morphological processes. These two model types are complementary:

(i) Mobile-bed river-models give easily bed-levels in two space dimensions. The length of the reach is restricted and time dependency is difficult to obtain.

(ii) Mathematical models of morphological processes are yet restricted to one space dimension (average values over the width of the rivers).

There is hardly a restriction as to the length of the river reach and time dependent values can be obtained.

This has led in the Netherlands to use a combination both types of models for tackling morphological problems (Sybesma et al, 1969).

4. Remarks

In this paper only some general considerations on design and operation of mobile-bed river-models are given. For many practical problems useful results can be obtained provided the investigator combines science and art! To improve the quality of the results, necessary due to the demand for increasing accuracy, first of all the fundamental knowledge on sediment transport and in particular on river morphology (in a quantirative sense) has to be increased. Moreover the experimental techniques can be improved. This, however, will lead to more expensive model-studies.

Here the common engineering sense has to give guidance. The accuracy of the model should not be better than absolutely necessary for the problem concerned. It is also impossible to get better results that the available prototype data permit.

This is again to stress the need of prototype data!

Symbols

C	Chézy coefficient	$[L^{\frac{1}{2}}T^{-1}]$
D	grain size	[L]
Fr	Froude number	-
h	depth of water	[L]
H	energy head	[L]
ΔH	difference in head	[L]
L	length	[L]
m	subscript for model	-
n_x	scale of parameter $x = L_p/L_m$	-

p	subscript for prototype	-
q_s	sediment transport (bulkvolume per unit of time and width)	$[L^2T^{-1}]$
Q_s	sediment transport (bulkvolume per unit of time)	$[L^3T^{-1}]$
r	distortion (factor)	-
R	radius of curvature stream line	$[L]$
S	slope	-
S_t	tilt	-
u	flow velocity	$[LT^{-1}]$
u_*	shear velocity	$[LT^{-1}]$
W	terminal fall velocity	$[LT^{-1}]$
Δ	relative density = $(\rho_s-\rho)/\rho$	-
μ	ripple factor	-
ρ	density water	$[ML^{-3}]$
ρ_s	density sediment	$[ML^{-3}]$

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